LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2025**

PMT1MC03 – ORDINARY DIFFERENTIAL EQUATIONS

Date: 28-04-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Answer the following**
a)	Define a first-order initial value differential equation and give an example.
b)	When do you say that two functions $x_1(t)$ and $x_2(t)$ are linearly dependent?
c)	Describe the systems of first-order equations.
d)	State the orthogonal property of Legendre polynomials.
e)	Provide an example of an oscillatory equation with an explanation.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**MCQ**
a)	Which of the following is the linear differential equation? (i) $x' + \lvert k\rvert x = 0$ (ii) $x' + k\lvert x\rvert = 0$ (iii) $x' + kx^2 = 0$ (iv) $(x')^2 + kx = 0$
b)	The Picard iteration formula is based on which fundamental mathematical concept? (i) Taylor series expansion (ii) Newton's method (iii) Integral equation (iv) Finite difference method
c)	A linear equation $x''' - 6x'' + 10x' - 6x = 0$ is transformed to a linear system $x' = Ax$, find A. (i) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & 10 & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & 10 & -6 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -10 & 6 \end{bmatrix}$ (iv) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -10 & -6 \end{bmatrix}$
d)	The recurrence relation for Bessel functions $J_p(t)$ is: (i) $J_{p+1}(t) = \frac{2p}{t}J_p(t) - J_{p-1}(t)$ (ii) $J_{p+1}(t) = J_p(t) - J_{p-1}(t)$ (iii) $J_{p+1}(t) = \frac{t}{2p}J_p(t) + J_{p-1}(t)$ (iv) $J_{p+1}(t) = J_p(t) + J_{p-1}(t)$
e)	Sturm's comparison theorem is primarily used to compare: (i) The growth rates of two functions (ii) The number of zeros of solutions to two differential equations (iii) The stability of two solutions (iv) The boundary conditions of two differential equations

	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Use the method of variation of parameters to find the solution of $x' + a(t)x = b(t)$ where *a* and *b* are known continuous function defined on the interval *I*.
4	If the Wronskian of two functions x_1 and x_2 on *I* is non-zero for at least one point of *I*, show that x_1 and x_2 are linearly independent. Apply this for the case $x_1 = sint, x_2 = cost,\ I = \mathbb{R}$,
5	Show that $\Phi(\mathrm{t}) = \begin{bmatrix} e^{-3t} & te^{-3t} & t^2e^{-3t}/2 \\ 0 & e^{-3t} & te^{-3t} \\ 0 & 0 & e^{-3t} \end{bmatrix}$ is a fundamental matrix for a linear system $x' = A(t)x$ where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$.
6	Using Bessel's function, show that $e^{\left(\frac{t}{2}(x-\frac{1}{x})\right)} = \sum_{n=-\infty}^{\infty} J_n(t)x^n$.
7	Let r_1, r_2 and p be continuous functions on (a, b) and $p > 0$. Assume that *x* and *y* are real solutions of $(px')' + r_1x = 0$ and $(py')' + r_2y = 0$ respectively on (a, b). If $r_2(t) \geq r_1(t)$ for $t \in (a, b)$, show that between any two consecutive zeros t_1, t_2 of *x* in (a, b), there exists at least one zero of *y* in $[t_1, t_2]$. What happens when $r_1 = r_2$?
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	Analyze the general criteria for ensuring the Lipschitz condition and illustrate with examples.
9	Let $b_1, b_2, \ldots, b_n : I \to \mathbb{R}$ be continuous functions in the *n*-th order homogeneous differential equation $L(x(t)) = 0$. Let $\varphi_1, \varphi_2, \ldots, \varphi_n$ be *n* linearly independent solutions of $L(x(t)) = 0$ on *I*. Derive the formula to the Wronskian of $\varphi_1, \varphi_2, \ldots, \varphi_n$.
10	Determine the two linearly independent solution of $x'' - 2tx' + 2x = 0$.
11	Explain the Hille-Wintner comparison theorem.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	Consider two types of living organisms that both rely on the same food source for survival. Formulate a mathematical model to describe this situation, and explain how the model could be used to predict the potential extinction of a particular species based on initial population sizes.
13	Let $x' = A(t)x$ be a linear system where $A: I \to M_n(R)$ is continuous. Suppose a matrix Φ satisfies the system, determine $(\det \Phi)'$ and discuss that Φ is a fundamental matrix if and only if $\det \Phi \neq 0$.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	Discuss the conditions for the existence of a unique solution for the first-order initial value problem $x' = f(t, x), x(t_0) = x_0$.
15	Derive the solution of the Legendre equation $(1 - t^2)x'' - 2tx' + p(p + 1)x = 0$ and determine the Legendre polynomial

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